Title: Geometric Mean Made Friendly

Brief Overview:

Using the TI-92, this lesson will demonstrate the geometric mean between two positive numbers using several methods.

Link to Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate the use of the TI-92 in solving problems concerning geometric mean.

- **Mathematics as Reasoning**
  Students will discover the formula for the relationship of cross-product values in the geometric mean based on observations.

- **Mathematical Connections**
  Students will relate the geometric mean to segments of a right triangle.

- **Number and Number Relationships**
  Students will demonstrate their ability to measure and compare lengths of segments and measures of angles.

- **Algebra**
  Students will solve rational equations for a variable.

- **Mathematics as Communication**
  Students will predict outcomes based on experimentation and will provide explanations for their predictions.

- **Geometry**
  Students will manipulate constructions and make conclusions based on observations. Students will use the vocabulary of geometry in the demonstration of the geometric mean.

- **Measurement**
  Students will use the TI-92 to measure segments and angles and to calculate the cross-product.

Grade/Level:

Grades 8-12

Duration/Length:

This activity will take 2 days of 50-minute periods or 1 block period. The activities may take longer than anticipated depending on class durations and students' prior knowledge and familiarity with the TI-92.

Prerequisite Knowledge:

Students should have working knowledge of the following:

- Identification of the parts of a right triangle
- Altitude
- Angle measurement
- Properties of similar triangles

Objectives:

Students will:

- develop cooperative learning skills.
- construct geometric figures using several techniques.
- investigate the geometric mean.
- collect and organize data using constructed and animated figures.
- save data to disk for report development.
- formulate hypotheses based on collected and observed data.

Materials/Resources/Printed Materials:

- TI-92 calculator
- Pencil/paper
- TI-92 overhead screen
- Computer
- TI-GRAPH LINK
- Low heat overhead projector
- Student worksheets
- Teacher's Guide

Development/Procedures:

- Divide the class into groups of two students and review basic vocabulary.

- Guide students through the constructions.

- Verify that there are three similar triangles in the construction.

- Use the TI-92 to show that the length of the altitude to the hypotenuse of a right triangle **cannot** be the geometric mean between the lengths of the legs of the triangle.

- Use the TI-92 to show that the length of the altitude from the right angle to the hypotenuse **is** the geometric mean of the lengths of the two segments of the hypotenuse.

- Use the TI-92 to show that if the altitude is drawn to the hypotenuse of a right triangle, then the length of either leg is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.

Performance Assessment:

Students will construct a right triangle, its altitude, measure segments, angles, record observations, drag the triangle using a vertex and record conclusions. Students will use constructions to verify that if a and b are positive numbers then positive number x is the geometric mean if and only if

$$\frac{a}{x} = \frac{x}{b}, \; x^2 = ab, \text{ and } \; x = \sqrt{ab}$$

Students will submit completed worksheets and/or word processor report containing TI-92 viewscreens of completed constructions with calculations.

Extension/Follow Up:

Use the geometric mean in application problems on the student worksheet.
Applications worksheet using geometric mean.

Authors:

Alan R. Brown
Frank W. Cox High School
Virginia Beach, VA

Julia V. Brown
Frank W. Cox High School
Virginia Beach, VA

Guided Constructions
Directions Provided by the Teacher

Construct a right triangle.

Construct a horizontal line along the lower edge of the viewscreen - F2:4.

Locate points A and B on this line using F2:2. It may be necessary to use 2nd CAPS (Z) to type the capital letters.

Construct a second line such that it will intersect the first line at point B - F2:4.

Construct a line perpendicular to the second line and through point A - F4:1.

Construct a point at the intersection near the top of the view screen - F2:2.

Construct triangle ABC - F3:3. The construction should be similar to figure 1.

Measure the acute angles of triangle ABC using three points to identify the angle - F6:3.

Show that these angles are complementary.

Construct a line perpendicular to line AB and through point C - F4:1.

Construct point D at the intersection of line CD and line AB - F2:3 (Figure 2)

Construct a segment on line CD from point C to point D - F2:5.

Construct segment AB on line AB.

Construct triangle ACD and triangle BCD - F3:3. (Figures 3 and 4)

Hide all lines - F7:1. (Figure 5)

Verify that the altitude is perpendicular to the hypotenuse - F6:8.
Delete the property using green diamond Z.

Measure the complementary angles: $\angle ACD$ and $\angle BCD$ - F6:3. Verify that these measurements are the same as in triangle ABC. Hide the measurements - F - 7:1.

The next phase of this activity is actual data collection which will be accomplished using the student worksheet. Each student group working independently will have slightly different data measurements yet should achieve the same conclusions.

Student Worksheet **Name: ______________________________ Bell _______**

Data Collection

Organize the data to be collected by using a series of comment boxes for the measure of all segments - F7:5. Measure each segment shown using F6:1. The measure should be taken "Distance from this point" enter "to that point"enter. After all measurements have been taken drag each measurement to its respective text label.

Record the measures of all segments and angles in the spaces below.

AB = _______ AC = _____ AD = _______

BC = ________ B D = _____________ CD = _______

$\angle A$ = ________ $\angle B$ = _____________ $\angle ACD$ = _______ $\angle BCD$ = _____

$\angle ADC$ = ______ $\angle BDC$ = ______ $\angle ACB$ = _______

Using F7:5 make comment boxes for each of the following on the viewscreen:
CD^2 = , AD*BD = , AC^2 = , AD*AB = , CB^2 = , DB*AB = .

Calculations

Use F6:6 and scroll through the left column to complete the algebraic expressions in the right column. For example: select AD and place the letter a in the calculate line by pressing enter. Press the multiplication symbol. Select the measure for BD and press enter. The letter b will be added to the calculate line after the * symbol. By pressing enter again the product will appear preceded by R for result. For best results, let the calculator perform the calculations. Greater accuracy will be achieved.

AD*DB = ______ **AD*AB = ______** **AB*BD = ______**

CD^2 = ______ **AC^2 = ______** **BC^2 = ______**

Observations and conclusions: (Use additional paper if necessary.)

Write proportions to explain the observed data.

a. b. c.

Use a word processor (Microsoft Works for Windows or WordPerfect 6.1 or 7) and the TI-GRAPH LINK to record each theorem and a figure which represents that theorem. If a word processor and/or graphlink are not available use notebook paper to record the theorem with an appropriate sketch of that theorem. In either event, copy the theorems provided here or use those which appear in your textbook.

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each. other.

Show that $\Delta ABC \sim \Delta ACD \sim \Delta CBD$. Write all combinations of the proportions and substitute measures obtained from the constructions. (Note: The measured values displayed are to two decimal places. The Calculate tool uses more of the value than is shown on the screen. If the displayed values are used in a calculator equality of cross-products can not be achieved. For best results, use the TI-92 for the calculations by scrolling through the selected values shown.) **Calculate the value of each ratio. Drag vertex C and note that the measures of the segments change but the ratios do not.**

1. All angles are ____________________.

2. $\Delta ABC \sim \Delta$ ______

   $\frac{\mathbf{AB}}{\mathbf{AC}}$ = _______ = _______

3. $\Delta ABC \sim \Delta$ ______

   $\frac{\mathbf{AB}}{\mathbf{CB}}$ = _______ = _______

4. $\Delta ACD \sim \Delta$ ______

   $\frac{\mathbf{AC}}{\mathbf{CB}}$ = _______ = _______

Student Worksheet
(Examples for Teacher Use)

Data Collection

Organize the data for the measurement of each segment using the following as an example. F6:1 Cursor to point A label, enter. Cursor to point B label, enter. After the number appears type AB =. The AB = will be shown preceding the distance measure. This text line can be dragged to a clear area of the viewscreen. Repeat this process for the other five measurements. (Figure 6)

Record the measures of all segments and angles in the spaces below.

AB = 5.06 cm AC = 3.91 cm AD = 3.01 cm

BC = 3.22 cm BD = 2.04 cm CD = 2.48 cm

$\angle A = 39.47^o$ $\angle B = 50.53^o$ $\angle ACD = 50.53^o$ $\angle BCD = 39.47^o$

$\angle ADC = 90.00^o$ $\angle BDC = 90.00^o$ $\angle ACB = 90.00^o$

Using F 7:5 make comment boxes for each of the following on the viewscreen next to the measurements: CD^2 = , AD*BD = , AC^2 = , AD*AB = , CB^2 = , DB*AB = .
(Figure 7)

Calculations

Use F6:6 and scroll through the left column to complete the algebraic expressions in the right column. For example: AD*BD will look like figure 8 which shows that AD has been selected and placed in the calculate line. The measure for BD is highlighted and once enter has been pressed the letter b will be added to the calculate line. By pressing enter again the product will appear preceded by R for result. For best results, let the calculator perform the calculations. Greater accuracy will be achieved.

AD*DB = 6.16 AD*AB = 15.25 AB*BD = 10.34
CD^2 = 6.16 AC^2 = 15.25 BC^2 = 10.34 (Figure 9)

Show that the length of the altitude squared does not equal the product of the legs of the right triangle. 6.16 12.56.

Observations and conclusions:

Write proportions to explain the observed data.

a. $\dfrac{AD}{CD} = \dfrac{CD}{BD}$

b. $\dfrac{AD}{AC} = \dfrac{AC}{AB}$

c. $\dfrac{AB}{BC} = \dfrac{BC}{BD}$

Use a word processor (Microsoft Works for Windows or WordPerfect 6.1 or 7) and the TI-GRAPH LINK to record each theorem and a figure which represents that theorem. If a word processor and/or graphlink are not available use notebook paper to record the theorem with an appropriate sketch of that theorem. In either event, copy the theorems provided here or use those which appear in your textbook.

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.

Show that $\Delta\, ABC \sim \Delta\, ACD \sim \Delta\, CBD$. Write all combinations of the proportions and substitute measures obtained from the constructions. (Note: The measured values displayed are to two decimal places. The Calculate tool uses more of the value than is shown on the screen. If the displayed values are used in a calculator equality of cross-products can not be achieved. For best results, use the TI-92 for the calculations by scrolling through the selected values shown.) **Calculate the value of each ratio. Drag vertex A or B and note that the measures of the segments change but the ratios do not.**

1. All Angles are congruent.

2. $\Delta\, ABC \sim \Delta\, ACD$

   $\dfrac{AB}{AC} = \dfrac{BC}{CD} = \dfrac{AC}{AD}$ $\qquad \dfrac{5.06}{3.91} = \dfrac{3.22}{2.48} = \dfrac{3.91}{3.01} = 1.30$

3. $\Delta\, ABC \sim \Delta\, CBD$

   $\dfrac{AB}{CB} = \dfrac{BC}{BD} = \dfrac{AC}{CD}$ $\qquad \dfrac{5.06}{3.22} = \dfrac{3.22}{2.04} = \dfrac{3.91}{2.48} = 1.57$

4. $\Delta\, ACD \sim \Delta\, CBD$

   $\dfrac{AC}{CB} = \dfrac{CD}{BD} = \dfrac{AD}{CD}$ $\qquad \dfrac{3.91}{3.22} = \dfrac{2.48}{2.04} = \dfrac{3.01}{2.48} = 1.21$

Geometric Mean Extensions

Copy these theorems into the word processor or into the notebook.

The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of its two segments of the hypotenuse.

If the altitude is drawn to the hypotenuse of a right triangle, then the measure of a leg of the triangle is the geometric mean between the measure of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

To emphasize the segment that is the geometric mean, use the bold or dotted line function F 7: 8 & 9, the animated function F 7:3 using a point on the segment that is the geometric mean, or use colored markers on an overlay. (Figures 10 - 12)

Geometric Mean Extended to Intersecting Chords of a Circle

Name: ________________________________ Bell: ______ Date: __________________

Draw a segment - F2:5.

Find its midpoint - F4:3.

Using the midpoint as the center construct a circle with the segment endpoints on the circle - F3:1.

Place a point anywhere between one endpoint and the center - F2:2.

Construct a line perpendicular to the circle diameter and through the random point - F4:1.

Place a point at the intersection of the perpendicular line and the circle - F2:3.

Measure the distance from the point on the circle to the random point on the diameter - F6:1.

Measure the distance from one end of the diameter to the random point - F6:1.

Measure the distance from the random point to the other end of the diameter - F6:1.

Show that the distance from the point on the circle to the random point is the geometric mean between the measures of the two segments that make up the diameter of the circle. This shows that if a and b are positive numbers and x is a positive number then

$$\frac{a}{x} = \frac{x}{b}, \qquad x^2 = ab, \qquad x = \sqrt{ab}$$

To reinforce the concept, construct the legs of the right triangle by drawing a segment from the endpoints of the diameter to the point on the circle.

Geometric Mean Worksheet

Name:______________________________ Bell: ____ Date: ___________

Use the relationships developed using the TI-92 and the textbook to solve each of the following.

Use the figure at the right and the given information to solve each problem.

1. **AD = 6 and DB = 12. Find CD**

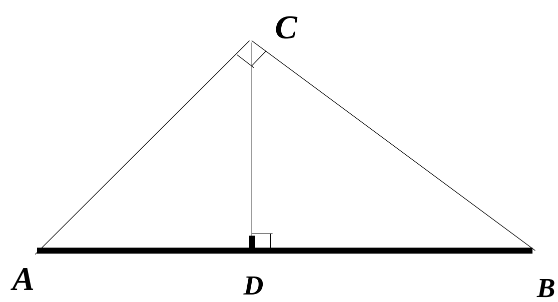
2. **Find CD if BD = 15 and AD = 3.**

3. **If AD = 12 and BD = 18, find CD.**

4. **Find AC if AB = 24 and AD = 3.**

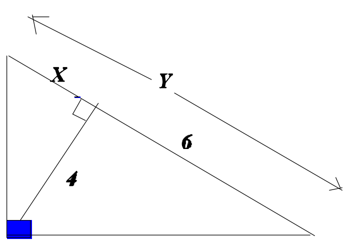
5. **Find CD if AB = 10 and AC = 6.**

6. **Find BC if AD = 4 and DB = 7.**



Find the values for the indicated variable.

7.



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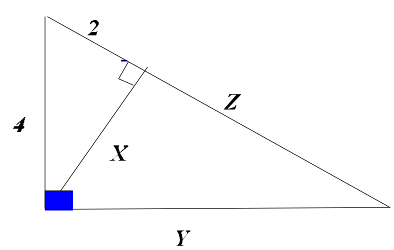

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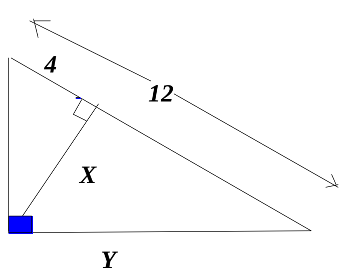

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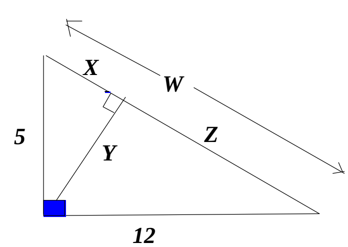

Find the geometric mean between the following:

11. **18 and 27**

12. **13 and 39**

13. **3 and 5**

14. **64 and 100**

15. $3\sqrt{3}$ **and** $5\sqrt{5}$

16. $5\sqrt{3}$ **and** $3\sqrt{5}$

17. $4\sqrt{3}$ **and** $5\sqrt{3}$

18. **4.8 and 12.8**

19. **The geometric mean is 16. The larger number is twice the smaller. What are the two numbers in simplified radical form?**

20. **The geometric mean is 48. The smaller number is one fourth that of the larger. What are the two numbers?**

Answers to Geometric Mean Worksheet

1. $6\sqrt{2}$ or 8.5
2. $3\sqrt{5}$ or 6.7
3. $6\sqrt{6}$ or 14.7
4. $6\sqrt{2}$ or 8.5
5. $AD = 3.6$, $BD = 6.4$, $CD = 4.8$
6. $2\sqrt{11}$ or 6.6
7. x = 2.7, y = 8.7
8. $x = 4\sqrt{2}$ (5.6), $y = 4\sqrt{6}$ (9.8)
9. $x = 2\sqrt{3}$, $z = 6$, $y = 4\sqrt{3}$
10. x = 1.92, z = 11.08, y = 4.62
11. $9\sqrt{6}$ or 22.0
12. $13\sqrt{3}$ or 22.5
13. 3.87
14. 80
15. 7.6
16. 7.6
17. 7.7
18. 7.8
19. $8\sqrt{2}$ or 11.3
20. 24

Geometric Mean Application Problems

21. To find the height of an apple tree in her yard, Susan held a clipboard near her eye so that the top of the tree was viewed along one edge of the book and the base of the tree was viewed along the adjacent edge. If her height of eye is 4.5 feet and she is standing 15 feet from the tree, how tall is the tree?

22. A mountain climber is standing one hundred feet from the base of a cliff. He sights along two edges of his climbers guidebook in order to view both the top and the bottom of the cliff. If his height of eye is 6 feet, how tall is the cliff?

23. A photographer wishes to set up her camera on a tripod that is 5 feet above the level ground. If the monument is 36 feet tall, how far from the monument should she place her camera?

Answers: 21. 54.5 feet 22. 1673 feet 23. 12.5 feet

Figures for Geometric Mean Made Friendly

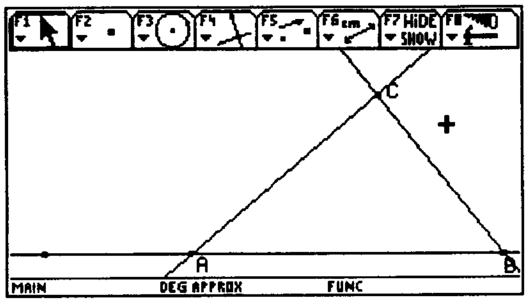

Figure 1

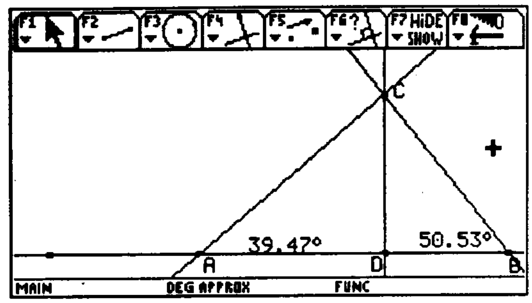

Figure 2

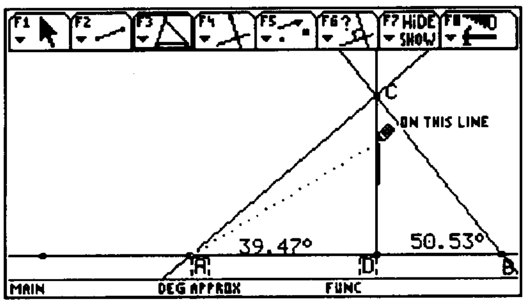

Figure 3

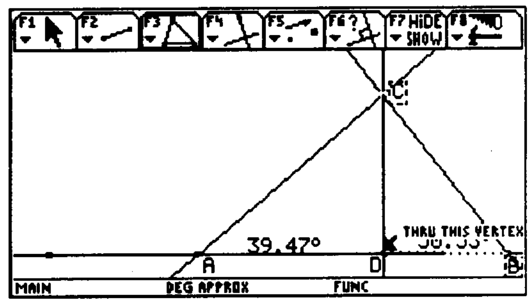

Figure 4

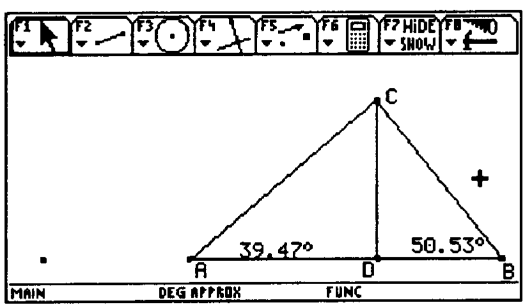

Figure 5

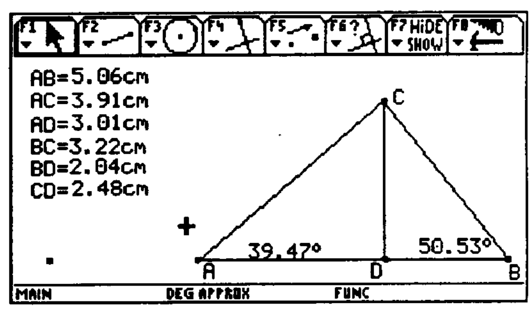

Figure 6

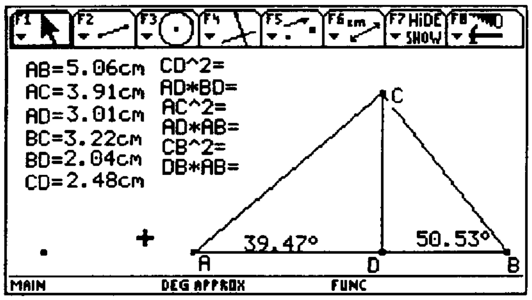

Figure 7

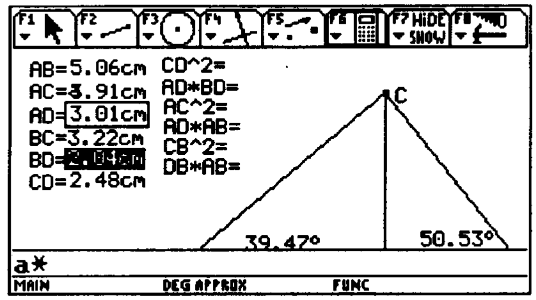

Figure 8

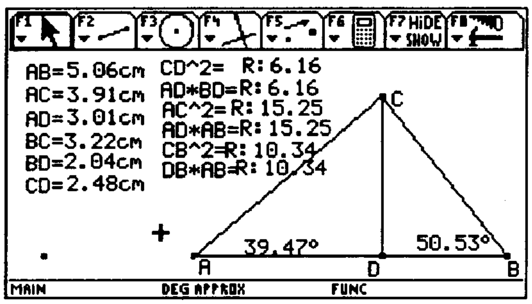

Figure 9

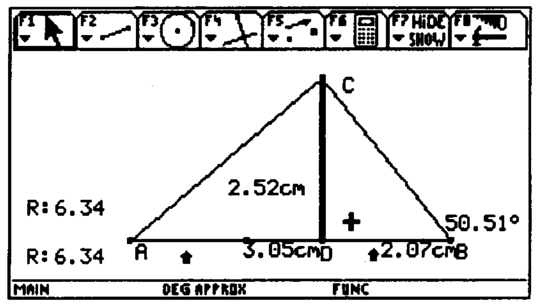

Figure 10

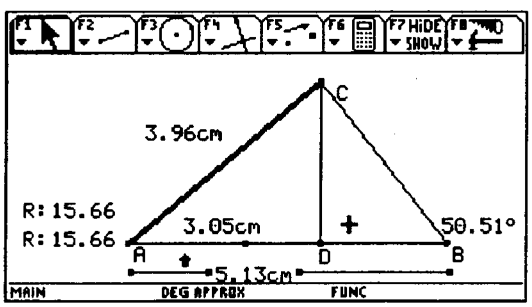

Figure 11

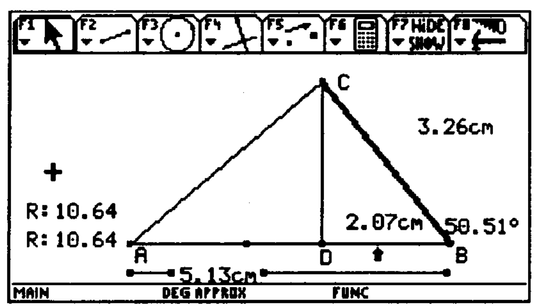

Figure 12